A Computer for *all* Students - Revisited

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Ten years ago graphing calculators started a revolution in the teaching and learning of mathematics. In early 1992 we published an article in the *Mathematics Teacher* describing inexpensive graphing calculators as computers with built-in graphing software (Demana and Waits 1992). These calculators could be viewed as computers available to all students because of their low cost, ease of use, and portability. Our point was that only a few elite would benefit if teachers had to rely exclusively on expensive computer laboratories to deliver computer-enhanced visualization in mathematics teaching and learning.

We followed the first article with another *Mathematics Teacher* article making a case against students' use of computer symbolic algebraic manipulation in school mathematics *at that time* (Waits and Demana 1992). Our reasons then were simple. Both software and hardware for computer algebra systems (CAS) were too expensive to be practical in most mathematics classrooms. We stressed that much of the use of CAS in grades 9–12 mathematics at that time involved producing numerical or graphical results that could be obtained more easily and much more practically with inexpensive graphing calculators! At the end of the "A Case Against . . . ." article we wrote, "We are convinced that CAS [computer algebra systems] methods are the way to go in the future. . . ."

THE FUTURE IS HERE!

Texas Instruments recently introduced a relatively inexpensive hand-held computer, the TI-92, including built-in computer symbolic algebra and computer interactive-geometry software. It is merely the first of a new generation of hand-held computers, since other calculator manufacturers will surely follow with similar products. These new hand-held personal computing tools contain very powerful and versatile computer software, and they truly represent a computer for *all* mathematics students.

IMPLEMENTING THE STANDARDS

A major concern to us today is that the following underlying assumptions about computer technology, as presented in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989, 124), are still very difficult to implement in typical high schools. The *Standards* document states, "A computer will be available at all times in every classroom for demonstration purposes, and all students will have access to computers for individual and group work."

One reason for our concern is the high cost of computer laboratories, their upkeep, and related teacher-in-service issues. Furthermore, the *Standards* technology strands for secondary school require students to use computer software that is far more sophisticated than what can be delivered by even the most modern graphing calculators. Dependence on only desktop computers and expensive software housed in computer laboratories is a major barrier to implementing serious technology-based curriculum reform in mathematics.

Many teachers simply opt to avoid using computer symbolic algebra and computer interactive geometry in their high school mathematics classes because it has not been practical or possible. Now it is both possible and practical! A classroom set of TI-92 hand-held computers is a much less expensive and portable computer laboratory. Implementing the important computer-technology strands of the NCTM's grades 9–12 Standards for *all* students is finally feasible.

A NEW CHALLENGE

Mathematics teachers have a new challenge. In the past, teaching traditional paper-and-pencil symbolic algebra skills in school mathematics was necessary because they were the only procedures available for the algebraic manipulations needed to solve problems. This lack of procedures no longer exists today. Computer symbolic algebra algorithms now do algebraic manipulations much faster and with far better accuracy than is possible with the traditional methods.

Our community can no longer ignore how students' use of computer symbolic algebra and computer interactive geometry affects the mathematics curriculum. This new generation of hand-held computing tools for students will become as popular as graphing calculators are today. We must deal with the fact that computer symbolic algebra and computer interactive geometry are better—far better—tools than paper and pencil for doing many of the manipulations associated with mathematics.

These new tools can also be used to better illustrate important concepts and applications of mathematics. We must redefine "basic skills" to include those paper-and-pencil manipulative tasks necessary to understand algebra as a language of representation. Some traditional paper-and-pencil skills will continue to be necessary for mathematical activities, as will traditional mental-mathematics skills. However, we must also agree to stop spending large portions of our time teaching obsolete paper-and-pencil algebra and calculus manipulations. These obsolete skills must be identified and deemphasized in the curriculum. Doing so is our challenge for the future.

We believe that what is needed now is a secondary mathematics curriculum that takes advantage of computer technology to assist students in becoming powerful and thoughtful “problem solvers.” Here are two examples. Consider everyone’s favorite algebra topic of factoring algebraic expressions. Factoring is still very important. After all, the fundamental theorem of algebra is a factoring theorem. This theorem is central to good mathematical understanding, as are the very important connections among factors, *x*-intercepts of the graph of functions, zeros of functions, and the behavior of functions. **Figure 1** shows the result of applying the straightforward "Factor(" CAS command of the TI-92 to a polynomial expression. The factors, by inspection, give us the solutions to the equation "expression" = 0 and tell us a great deal more information about the behavior of the function than does the nonfactored form.

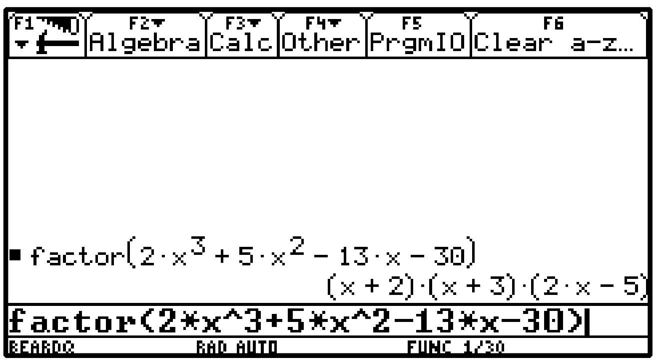

Fig. 1
A CAS factoring example

Another example, from calculus, is the use of partial fractions as an integration technique. **Figure 2** shows the result of applying the "Expand(" CAS command to a rational function $g(x)$. Notice that the technology now makes it possible to integrate $g(x)$ by finding elementary antiderivatives mentally!

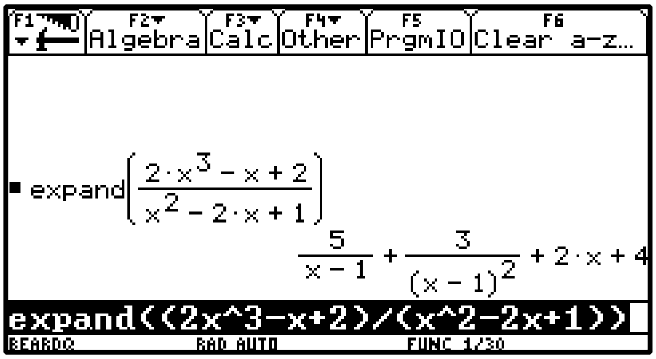

Fig. 2

A CAS partial fraction decomposition

The result of applying the "Integrate" CAS command directly before the CAS partial fraction decomposition is shown in **figure 3,** just to check our mental-integration skills!

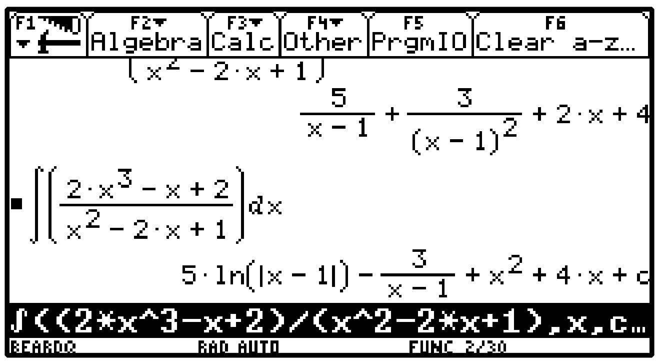

Fig. 3

An indefinite-integral CAS computation

We believe that teachers should definitely continue to teach factoring and partial fraction decomposition, as well as what those topics mean and why the related skills are useful. However, the tools that people will use to factor and to find partial fraction decompositions will move from paper and pencil to computer symbolic algebra tools. Teachers should teach these old standard and comfortable topics but should spend far less time with paper-and-pencil methods and far more time with CAS tools. The pedagogical thrust should be not to delete traditional topics but to reduce the time spent and change the tools used for these topics.

Educators need to have textbooks and tests that reflect use of these new technology tools and thus better represent the new curriculum. The education community needs to find appropriate ways to support and fund teachers who need in-service training. Teachers will find that they need to become increasingly more technology literate and will need to become curriculum reformers as well.

A MATHEMATICS IMAGE PROBLEM

The mathematics community must do a better job of addressing the national “mathematics image problem.” The public often associates “doing mathematics” with only the mental and paper-and-pencil arithmetic and algebraic computations and manipulations they learned in school. As educators, we need to communicate convincingly to the general public that “doing mathematics” in the twenty-first century means much more than doing the mathematics of the past. School mathematics in the future will be far more technologically enhanced, richer, more interesting, and more applicable than in the past. Business and industry want employees who can think, read, and understand problem situations; work cooperatively in groups; understand and use technology; and communicate effectively with others. The appropriate use of technology in mathematics teaching and learning helps build these important skills in students.

SUMMARY

Calculators with built-in graphing software for enhancing mathematics teaching and learning are now over ten years old. Casio invented and marketed the first graphing calculator in 1985 and started a revolution in delivering powerful computer graphing to millions of mathematics students. Affordable graphing calculators have certainly fulfilled our dream of making it possible for *all* mathematics students to use computer visualization on a regular basis for both in-class and out-of-class activities—a dream that could never have been realized with desktop personal computers in computer laboratories.

Our world of mathematics teaching and learning will never be the same. We believe that Texas Instruments has fired the first shot in the next revolution of hand-held computer technology designed for use in school mathematics. Surely similar products will be available from other calculator companies in the future.

The NCTM’s *Curriculum and Evaluation Standards for School Mathematics* (1989) speaks well to the content topics and methods that are needed in a modern mathematics curriculum for all students. Today a new tool is available that makes possible and practical the vision of the rich computer-technology-enhanced mathematics curriculum, for *all* students, represented by the NCTM's *Standards* document. By facilitating the reduction of time spent on paper-and-pencil methods, this tool will permit additional time for the new mathematical content and activities recommended in the *Standards.*

Educators need to be more specific and explicit about a controversial issue. We can no longer spend mathematics classroom time doing everything we did in the past paper-and-pencil era and covering the many new topics and methods that our students need for the technologically intensive future they face. Let us all get busy today and agree on what paper-and-pencil computation and manipulation algorithms can be done better by computers. Our students are waiting for us!

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